Spatial and spectral evolution of turbulence[a)]

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Spreading of turbulence as a result of nonlinear mode couplings and the associated spectral energy transfer is studied. A derivation of a simple two-field model is presented using the weak turbulence limit of the two-scale direct interaction approximation. This approach enables the approximate overall effect of nonlinear interactions to be written in the form of Fick's law and leads to a coupled reaction-diffusion system for turbulence intensity. For this purpose, various classes of triad interactions are examined, and the effects that do not lead to spreading are neglected. It is seen that, within this framework, large scale, radially extended eddies are the most effective structures in promoting spreading of turbulence. Thus, spectral evolution that tends toward such eddies facilitates spatial spreading. Self-consistent evolution of the background profile is also considered, and it is concluded that the profile is essentially slaved to the turbulence in this phase of rapid evolution, as opposed to the case of avalanches, where it is the turbulence intensity that would be slaved to the evolving profile. The characteristic quantity describing the evolving background profile is found to be the mean "potential vorticity" (PV). It is shown that the two-field model with self-consistent mean PV evolution can be reduced to a single Fisher-like turbulence intensity transport equation. In addition to the usual nonlinear diffusion term, this equation also contains a "pinch" of turbulence intensity. It is also noted that internal energy spreads faster than kinetic energy because of the respective spectral tendencies of these two quantities. © *2007 American Institute of Physics.*
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I. INTRODUCTION

Motivation

It is a common observation that a localized patch of sufficiently developed turbulence tends to spread and entrain laminar or less turbulent regions when left to its own devices (see Fig. 1). This is a result of the "turbulent action" itself. In simple terms, the "whirling motion" (i.e., mixing) due to turbulence causes the turbulence intensity to be spatially redistributed and homogenized. Simple as it may sound, a complete description of this observation requires a full understanding of the turbulent phenomena. Hence, simplified "thermodynamic" models[1] in the form of familiar $K-\epsilon$ or $K-\omega$ models that describe the evolution of macroscopic observables in turbulent evolution are frequently used in engineering applications. Moreover, since most of the analytical works on turbulence theory deal with spectral instead of spatial evolution—and boldly assume homogeneous and isotropic turbulence—they are not of much help in this problem. Nevertheless, self-similarity, which is at the heart of these analytical works and of spectral evolution, is still a solid foundation upon which one can build.

The spreading phenomenon of "mixing of turbulence by turbulence itself" consists of various identifiable ingredients. First of all, assuming the turbulence is driven in a localized region in space, it is possible to distinguish "leaking" of turbulence intensity from the unstable region (where the turbulence is driven) into the stable region (where the free-energy source is absent). This is in a sense "the first step" and is what we usually mean by turbulence spreading. Then this "leaking" turbulence in the stable region causes the stability boundary itself to be modified (sometimes called "penetrative convection"). Here we consider the case in which the stability boundary moves *as a result of turbulence spreading* and the resulting Reynolds stresses. The opposite case in which the turbulence evolves as a result of the evolution of the profile is usually associated with avalanches. Note also that for Kelvin-Helmholtz type instabilities such as the one shown in Fig. 1, the free-energy source is the sheared flow itself. This makes it slightly more difficult to separate these two ingredients.

Here we will discuss both of these ingredients and introduce a self-consistent model of intensity/profile evolution that takes into account both the turbulence intensity and the mean density or temperature profile. In particular, we will use a two-field physical model, the Hasegawa-Wakatani model, and recognize that the characteristic degrees of freedom for the self-consistent model are the kinetic energy spectrum, the internal energy spectrum, and the mean potential vorticity profile. Notice that for a mean-field Hasegawa-Wakatani system, the radial gradient of mean vorticity [i.e., $\nabla^2\bar{\Phi}(x)$] may also act as a free-energy source for the fluctuations as well as the radial gradient of mean density [i.e.,

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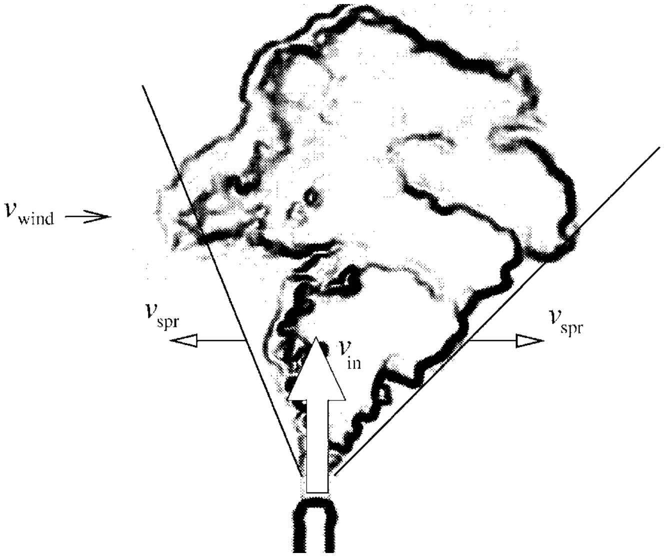

FIG. 1. Turbulence spreading observed in a smoke over a chimney. It is curious that a straight line is almost a perfect fit for the expansion of the boundary.

$\bar{n}(x)$], and thus combining the two in the form of potential vorticity is both highly desirable and natural.

In the context of magnetically confined plasmas, turbulence spreading was initially pointed out by Garbet *et al.*[2] in a detailed study, which compared the efficacy of spreading via nonlinear coupling with that via linear coupling of poloidal harmonics due to toroidicity effects. They concluded that while nonlinear effects tend to result in diffusive spreading, toroidicity effects lead to convective spreading. It should be noted that in this work, the nonlinear case examined was one of strong turbulence without linear growth and nonlinear damping.

Meanwhile, following a surge of interest in avalanches and self-organized criticality, a nonperturbative bivariate Burger's equation model of transport of turbulence intensity and profile evolution was proposed.[3–5] This constitutes a simple model of the spatial and spectral evolution of turbulence intensity, applicable both to the internal transport barrier (ITB) formation problem as well as to spreading. Note that the spatial evolution in this model involves the radial direction while the spectral evolution focuses on the poloidal wave number k_θ.

The current interest in the subject was sparked by observations of turbulent fluctuations in locally stable or damped regions of both simulations[6,7] and physical experiment.[8] Recently, a Fokker-Planck-type model of intensity transport was applied to the spreading problem.[9–11] This model, which was very much in the vein of $K-\epsilon$ models of fluid turbulence,[12] described the evolution of $\varepsilon(x,t)$, the turbulence intensity field, using a reaction diffusion equation similar to the well-known Fisher equation.[13,14] It was shown that an exact solution of this model exists,[10,15] which is a propagating ballistic front at a speed given by the geometric mean of diffusion and linear growth [i.e., $v=[\gamma D(\varepsilon)/2]^{1/2}$, where γ is the growth rate and $D(\varepsilon)$ is the turbulent diffusion coefficient of turbulence intensity]. Recently, self-consistent pro-

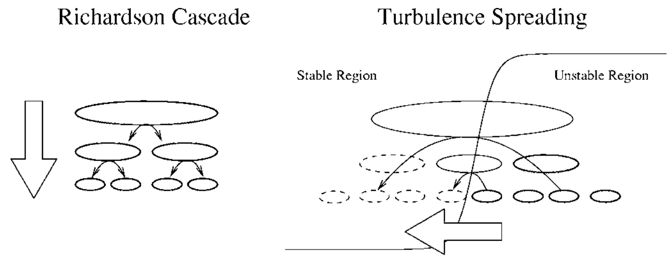

FIG. 2. The comparison of Richardson cascade and turbulence spreading via mode couplings in two dimensions related to dual cascade.

file evolution within this simple model has been studied in one and two dimensions,[16] confirming these basic tendencies. Also, a two-field version was derived, where various classes of triad interactions were systematically studied[15,17] and it was concluded that large-scale streamers (i.e., $k_x \sim k_z \sim 0$, $k_y\rho_s \ll 1$ modes) are the most efficient in causing the turbulence to spread (see Fig. 2). This also suggests that in an inhomogeneous three-wave interaction picture, zonal flows, which shear apart these structures, do not enhance but rather diminish spreading in accord with the familiar intuition about zonal flows. When zonal flow damping is included, however, spreading results due to the damping of the zonal flows. Recent numerical simulations[18] suggest that this is the case also when the damping on the zonal flows is collisionless or externally imposed. Note that the model that we suggest here takes into account the evolution of the potential vorticity (PV) profile and a mean PV gradient may generate local "linear" instability either via the usual drift instability or via a Kelvin-Helmholtz instability. This type of instability of the mean PV profile includes elements from collisionless (turbulent) damping of the mean flows. We argue that it is important to distinguish these and the "zonal" flows.

It has also been suggested that turbulence spreading is linked to the breaking of the gyro-Bohm scaling observed in computer simulations.[19–21] In this context, it is also useful to identify a range of length scales associated with the nonlocal dynamics of turbulence spreading. Despite recent progress in the area, the current understanding of the fundamental dynamics of turbulence spreading as a result of nonlocal, nonlinear mode couplings is still not satisfactory. In particular, the relation between the mechanism of spreading and those of nonlinear wave interaction processes in drift wave turbulence is not well understood.

Here we present a simple model of nonlinear turbulence spreading derived using the paradigm of weak wave turbulence. This is in contrast to the classical problem of the spectral evolution of fully developed turbulence or the formation of the linear eigenmode structure. The model is based on a gradient diffusion hypothesis derived rigorously from a statistical closure based on resonant three-wave interactions in an inhomogeneous background. We also consider the case in which the background profile self-consistently evolves with the turbulence, and we show that during the rapid evolution corresponding to turbulence spreading, the tendency to ballistic spreading also extends to the profile. Here the mean PV is used in order to describe the evolution of the background profile[22] instead of density or mean electric field, since total PV (i.e., $q = \ln n - \nabla^2\Phi$) is exactly conserved by the Hasegawa-Wakatani dynamics. This allows the local "linear" instabilities of the mean flow to be incorporated into the self-consistent profile evolution conveniently. Note that one can write for the Hasegawa-Wakatani system by subtracting the mean vorticity equation from the mean density equation,

$$D_t\bar{q} + \nu\nabla^2\bar{q} = \nabla \cdot \langle(\hat{\mathbf{z}} \times \nabla\tilde{\Phi})\tilde{q}\rangle. \tag{1}$$

In view of the weak turbulence approximation, we consider only resonant interactions. We show that it is possible, within this framework, to systematically study the mechanism of spreading in weak, wave turbulence and to assess the relative importance of different classes of wave-wave interactions. This is accomplished by first examining the general structure of the intensity flux and then constructing a transport model for the multicomponent fluctuation energy density.

Note that a similar formulation could be extended to the strong turbulence case. Our experience with strong turbulence suggest that the resulting "renormalized" model would have the same form as the weak turbulence case, with nonlinear damping and nonlinear diffusion terms being proportional to the turbulence amplitude instead of intensity.

II. METHOD AND FORMULATION

A. Turbulence dynamics

Tokamak plasmas usually have weak wave turbulence. This is manifested by the scaling of the saturation level and the diffusion coefficients with intensity rather than amplitude (i.e., $D \sim |e\Phi/T_e|^2$ instead of $D \sim |e\Phi/T_e|$). Since the waves in magnetically confined systems are necessarily anisotropic, wave turbulence also tends to be anisotropic. Moreover, the turbulence spreading problem inevitably deals with inhomogeneous turbulence. As a result, the successful paradigm of fully developed turbulence, where the coupling of small scales to the large scales is modeled as an effective eddy diffusivity, can no longer be employed. Instead, we use a simple paradigm based on scale-separation and two-scale resonant interactions.

The first step in the formulation of this problem is to compute the fluxes of turbulence kinetic and internal energy induced by three-wave interactions. In order to do this, we use a Markovian two-scale direct interaction approximation (TSDIA),[23] assuming weak turbulence and weak mean flows. Here the fact that the turbulence is weak allows us to compute the turbulent energy fluxes using only the resonant three-wave interactions. The formulation of the model is based on scale separation [i.e., $\Phi = \Sigma_{\mathbf{k}}\Phi_{\mathbf{k}}(X)e^{i\mathbf{k}\cdot\mathbf{x}}$] and requires computing the fluxes of nonlinear kinetic (i.e., $K = \langle|\nabla\Phi|^2\rangle$) and internal energy (i.e., $N = \langle n^2\rangle$) in the radial direction. These are roughly similar in various drift wave turbulence models. The fluxes are

$$\Gamma_K = \frac{1}{2}\langle\Phi^2\hat{\mathbf{z}} \times \nabla(\nabla^2\Phi)\rangle_X = \mathrm{Re}\left[\frac{i}{6}\sum_{\mathbf{p}+\mathbf{q}+\mathbf{k}=0}(q_y q^2 + p_y p^2 + k_y k^2) \times \langle\Phi_{-\mathbf{k}}\Phi_{-\mathbf{q}}\Phi_{-\mathbf{p}}\rangle\right], \tag{2}$$

$$\Gamma_N = -\left\langle\frac{n^2}{2}\partial_Y\Phi\right\rangle = -\mathrm{Re}\left[\frac{i}{6}\sum_{\mathbf{p}+\mathbf{q}+\mathbf{k}=\mathbf{0}}(k_y\langle\Phi_{\mathbf{k}}n_{\mathbf{p}}n_{\mathbf{q}}\rangle + p_y\langle\Phi_{\mathbf{p}}n_{\mathbf{q}}n_{\mathbf{k}}\rangle + q_y\langle\Phi_{\mathbf{q}}n_{\mathbf{k}}n_{\mathbf{p}}\rangle)\right]. \tag{3}$$

These are derived by substituting the Fourier expansions into the expressions for flux and averaging. Notice that the factor $1/6$ is the result of writing the permutations of wave numbers explicitly.

Various observations can be made from this general form of the flux. First, close to the adiabatic limit (i.e., $k_\parallel^2 v_{the}^2 L_n/c_s > 1$), where the dispersion relation for drift waves is $\omega_{\mathbf{k}} \approx k_y/(1+k^2)$, the kinetic energy flux coefficient $\Lambda_{kpq} \equiv (q_y q^2 + p_y p^2 + k_y k^2)$ vanishes for longer wavelengths $k \ll 1$ (i.e., for the most interesting limit for drift waves) whenever the three-wave resonance condition is satisfied (i.e., $\Delta\omega_{kpq} = \omega_{\mathbf{k}} + \omega_{\mathbf{p}} + \omega_{\mathbf{q}} = 0$ and $\mathbf{p}+\mathbf{q}+\mathbf{k}=0$). Thus, somewhat surprisingly, the kinetic energy cannot "spread" itself in the adiabatic (i.e., Hasegawa-Mima) limit. This suggests that three-wave interactions in drift wave turbulence possess an element of resiliency or self-binding. Second, if we pick one of the modes as a zonal flow (i.e., $q_y = 0$), the kinetic energy coefficient vanishes regardless of the collisionality limit whenever there is resonance [i.e., $\Lambda_{kpq} \sim k_y(k^2 - p^2) \sim \Delta\omega(1+k^2)(1+p^2)$]. However, this is not true for "streamers," which are radially elongated structures (i.e., $q_x = 0$) that are effective in mixing the turbulence in the radial direction since their flow is in the radial direction. We can derive the fluxes by the statistical closure technique, and the result has the form of Fick's law,

$$\begin{bmatrix}\Gamma_{\mathbf{p}}^{(K)} \\ \Gamma_{\mathbf{p}}^{(N)}\end{bmatrix} \equiv \begin{bmatrix}D_{\mathbf{p}}^{KK} & D_{\mathbf{p}}^{KN} \\ D_{\mathbf{p}}^{NK} & D_{\mathbf{p}}^{NN}\end{bmatrix}\partial_X\begin{bmatrix}K_{\mathbf{p}} \\ N_{\mathbf{p}}\end{bmatrix} \tag{4}$$

resembling the flux-force relation form from the collisional transport theory of gases. However, we note that here $D^{\alpha\beta}$ has off-diagonal components that are not positive definite and that $D^{KK} \neq D^{NN}$, in general. Clearly, D^{NN} is "closer" to the turbulent diffusion of a passive scalar than D^{KK}, which is related to the advection of $\nabla^2\Phi$ and therefore very nonpassive. In (4), the elements of the transport matrix are

$$D_{\mathbf{p}}^{(KK)} \equiv \frac{1}{2}\int d^2\mathbf{k}\int d^2\mathbf{q}\frac{(k_y k^2 + p_y p^2 + q_y q^2)}{k^2 p^2 q^2} \times \delta(\mathbf{k}+\mathbf{p}+\mathbf{q})[\delta(\Delta\omega^+)+\delta(\Delta\omega^-)] \times \{[q_y(q^2-p^2)+2p_x\hat{\mathbf{z}}\times\mathbf{q}\cdot\mathbf{p}]K_{\mathbf{q}} + [k_y(k^2-p^2)-2p_x\hat{\mathbf{z}}\times\mathbf{q}\cdot\mathbf{p}]K_{\mathbf{k}}\}, \quad (5)$$

$$D_{\mathbf{p}}^{(NN)} \equiv \frac{1}{2}\int d^2\mathbf{k}\int d^2\mathbf{q}\left(\frac{q_y^2}{q^2}K_{\mathbf{q}}+\frac{k_y^2}{k^2}K_{\mathbf{k}}\right)\times\delta(\mathbf{k}+\mathbf{p}+\mathbf{q}) \times[\delta(\Delta\omega^+)+\delta(\Delta\omega^-)], \quad (6)$$

$$D_{\mathbf{p}}^{(NK)} \equiv -\frac{1}{2}\int d^2\mathbf{k}\int d^2\mathbf{q}\left(\frac{q_y p_y}{p^2}N_{\mathbf{q}}+\frac{k_y p_y}{p^2}N_{\mathbf{k}}\right) \times\delta(\mathbf{k}+\mathbf{p}+\mathbf{q})[\delta(\Delta\omega^+)+\delta(\Delta\omega^-)], \quad (7)$$

and $D_{\mathbf{p}}^{(KN)}=0$. Here $\Delta\omega^+$ is the mismatch between frequencies of three growing modes and $\Delta\omega^-$ is the mismatch when one of the modes is damped. The detailed derivation of Eqs. (5)–(7) can be found elsewhere.[15] The key steps in the derivation are the assumptions of two-scale evolution for the "beat mode" in the DIA, which leads to the Fick's law form, and weak turbulence (also RPA), which results in the δ functions that impose the resonance conditions $\Delta\omega^{\pm}=0$. Notice that in the case of the near-adiabatic limit, the damped mode is strongly damped [i.e., $\gamma^- \sim -c(1+k^2)/k^2$], so in this limit $\Delta\omega^-$ can be neglected. It is also useful to note here that we have neglected resonance broadening in the transport matrix, consistent with weak turbulence, and higher-order nonlinear effects corresponding to nonlinear corrections to radial group velocity. Thus, the "convective term" ($\Gamma \sim V^{\alpha\beta}N^{\beta}$) consisting of these two higher-order corrections is neglected. Here $D^{(KK)}\propto K$ is the nonlinear self-diffusion of kinetic energy, $D^{(NN)}\propto K$ is the nonlinear diffusion of internal energy by the drift motions, and $D^{(NK)}\propto N$ is a radial stress acting on the local internal energy (and so is an off-diagonal term).

B. Profile dynamics

Strictly speaking, "spreading" (or turbulence overshoot) means leaking of turbulence into stable regions. However, as a result of this, the profile itself—whose gradient is the free-energy source—may also be modified (i.e., "penetrative convection"). The instability for the case of the Hasegawa-Wakatani system is due to the background density gradient and electron nonadiabaticity. This suggests that an evolving mean density should also be considered. However, when the mean fields are introduced, the evolution of the fluctuations is also modified, and the "local" dispersion relation can be written as

$$\omega_k'^2 + i\omega_k'\left[2i\nu k^2 + ic\left(\frac{1}{k^2}+1\right)\right] - \nu^2 k^4 - \nu c(1+k^2) + i\frac{c}{k^2}\hat{\mathbf{z}}\times\nabla\bar{q}\cdot\mathbf{k} = 0.$$

This is the usual Hasegawa-Wakatani dispersion relation with $\omega_k' = \omega_k - \overline{\mathbf{V}}\cdot\mathbf{k}$ and $\hat{\mathbf{y}}dn_0/dx$ replaced by $\hat{\mathbf{z}}\times\nabla\bar{q}$, where $\bar{q}\equiv n_0(x)+\bar{n}-\nabla^2\bar{\Phi}$ is the potential vorticity. Here ν is the model kinematic viscosity, $\chi=\nu$ is the particle diffusivity, and c is the collisionality parameter. The growth rate

$$\gamma_k(\bar{q}) \sim \frac{1}{c}\frac{\omega_k'^{(r)}(\omega_k'^{(r)} - \hat{\mathbf{z}}\times\nabla\bar{q}\cdot\mathbf{k})k^2}{(1+k^2)^3} - \nu k^2$$

suggests that when both mean density and mean flow are allowed to evolve dynamically, local drift instabilities appear, for which the mean PV gradient acts as the local source of free energy. [Note that $\omega_k'^{(r)} \sim \hat{\mathbf{z}}\times\nabla\bar{q}\cdot\mathbf{k}/(1+k^2)$ also depends on PV.] This can be modeled by a "local" mean growth rate

$$\bar{\gamma}(X) \approx -\alpha\partial_X\bar{q}(X), \quad (8)$$

where mean PV evolution given by (1) can be written using a simple quasilinear closure as

$$\partial_t\bar{q} \sim \partial_X(\beta D_{q0}\partial_X\bar{q}) - \partial_X(V_q\bar{q}\varepsilon). \quad (9)$$

Here $\varepsilon=N+K$, and V_q, β, and D_{q0} are parameters of the model. Note that we shall further take $D_{q0}=0$ in order to isolate rapid profile evolution due to spreading as opposed to usual collisional or quasilinear diffusion (e.g., $D_{q0}\sim\varepsilon$) related to the particle and momentum transport. In this limit, the PV profile would be absolutely stationary in the absence of spreading. Since spreading is usually faster than particle transport, this is a reasonable approximation for the spreading phase. Also, note that V_q has the dimensions of velocity and corresponds to a PV "pinch" velocity.

III. THE MODEL

The general two-field model using the computed fluxes consists of the evolution of kinetic energy,

$$\frac{\partial}{\partial t}K + v_{gx}\frac{\partial}{\partial x}K - \frac{\partial}{\partial x}\left(D_1 K\frac{\partial}{\partial x}K\right) = \bar{\gamma}(x)[\beta N + (1-\beta)K] - \gamma_{\mathrm{NL}}K^2 \quad (10)$$

and the internal energy,

$$\frac{\partial}{\partial t}N + v_{gx}\frac{\partial}{\partial x}N - \frac{\partial}{\partial x}\left(D_2 N\frac{\partial}{\partial x}K\right) - \frac{\partial}{\partial x}\left(D_3 K\frac{\partial}{\partial x}N\right) = \bar{\gamma}(x)[\beta K + (1-\beta)N] - \gamma_{\mathrm{NL}}N^2. \quad (11)$$

Here the diffusion coefficients are ordered as $D_3 > D_2 > D_1$ since this is the case for the overwhelming majority of the wave numbers. In (10) and (11), $\bar{\gamma}(x)$ is the self-consistent "local" mean growth rate calculated using (8) and (9).

One important point to note here is the fact that this effectively three-field model can in fact be reduced to the previous one-field model[9,10] in the proper limit. The limit corresponds to $V_\mu=0$, $D_{n0}=0$, and $K\sim N\sim\varepsilon/2$, where we define $D_0\equiv(D_1+D_2+D_3)/4$. However, for finite V_μ a different single-field equation can be derived, incorporating both overshoot and penetrative convection. This can be done by separating $\bar{q}=q_0+\delta\bar{q}$ and "solving" (9) for $\delta\bar{q}$ and then substituting the result into (8) assuming slow spatial evolution. This gives

$$\bar{\gamma} \sim \gamma_0(x)\left(1 - 2\tau V_\mu \frac{\partial \varepsilon}{\partial x}\right).$$

Using this as the local mean growth rate, we obtain

$$\frac{\partial}{\partial t}\varepsilon(x) + v_{gx}\frac{\partial}{\partial x}\varepsilon(x) - \frac{\partial}{\partial x}\left[D_0\varepsilon(x)\frac{\partial}{\partial x}\varepsilon(x)\right] - \gamma_0(x)\varepsilon(x)\left(1 - 2\tau V_\mu \frac{\partial \varepsilon(x)}{\partial x}\right) = -\gamma_{\rm NL}\varepsilon(x)^2. \quad (12)$$

This is a Fisher-Burger equation, with a nonlinear "advection" term. This is related to both the 1D Fisher equation model (e.g., Ref. 10) and the bivariate Burger equation model.[4] The solutions of this equation are propagating fronts, with basic spreading velocity [i.e., $v \sim (D_0\gamma\varepsilon_0/2)^{1/2}$] modified on both sides by the advection velocity (roughly added on one side, subtracted on the other). For $V_\mu > 0$, nonlinear velocity accelerates the front moving to the right, while slowing down the front moving to the left. Local saturation suggests the basic spreading velocity dominates over the Burger velocity as long as

$$D_0 > 2V_\mu \tau \Delta x \gamma_0(x),$$

which is usually true since $\gamma_0\tau \ll 1$. Note that here, τ is the response time for mean PV. The linear growth rate of the mean flow can be used in place of τ. Note that, if the previous single-field equation (e.g., Ref. 10) is a quasilinear transport equation for the turbulence spectrum, then (12) is a quasilinear transport equation, which also includes the turbulence intensity "pinch."

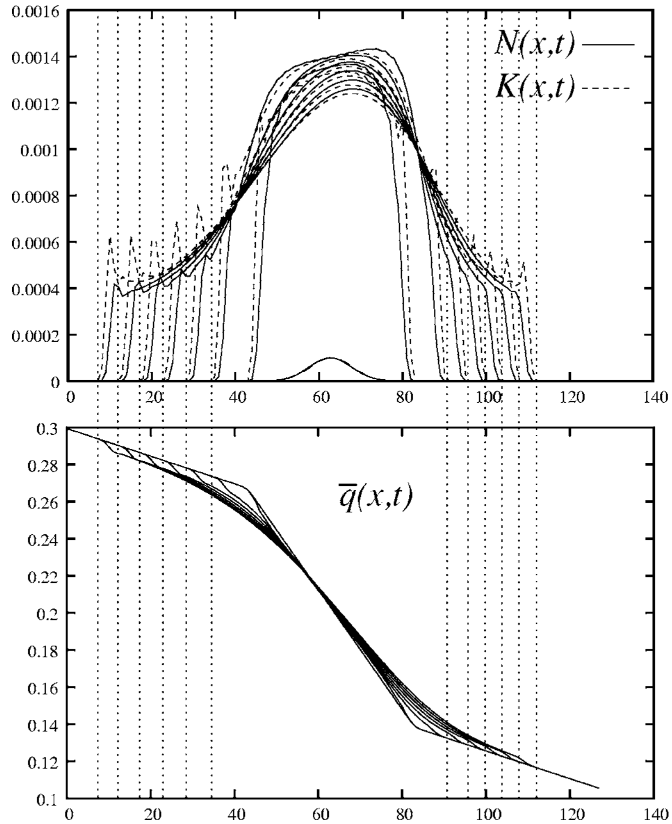

FIG. 3. Self-consistent evolution of internal energy N, kinetic energy K, and mean potential vorticity profile $\bar{q}$. It is possible to see that N leads both $\bar{q}$ and K as expected. Since the snapshots are taken at equal time intervals, spreading can be seen to be ballistic.

IV. RESULTS AND CONCLUSIONS

The basic results of the numerical integration of the three-field model of the previous section can be found in Fig. 3. It is clear from this figure that turbulence spreading is ballistic even with a self-consistently evolving profile and can be described using a Fisher-Burgers equation within reasonable approximations. Left-right asymmetry, which can be observed in the figure, is the result of the pinch effect described in the previous section when the Fisher-Burgers equation model was introduced.

Even though turbulence spreading and coupled profile evolution is a major issue relevant for avalanches and other bursty mesoscale phenomena such as rapid profile evolution, we believe that it is counterproductive to start from turbulence spreading and try to construct a complete self-consistent transport model. On the other hand, we believe that it is useful to include turbulence intensity as another degree of freedom in existing transport codes. We argue that transport of the turbulence intensity profile can be driven by the background density or temperature profiles, just as the particle or heat transport can be driven by the deposition profiles. In addition, we have shown here that the basic effect of turbulence driven particle pinch on the profile leads self-consistently to a "turbulence pinch" in the intensity evolution. Note that, even though "convective" transport of turbulence intensity, resulting from mode coupling, is not explicitly considered (the direct effect is higher order within the framework of inhomogeneous three-wave interactions), self-consistent evolution resulting in a simple Fisher-Burgers equation model has convective as well as diffusive behavior.

We also want to point out that it has been suggested that zonal flows may "promote" spreading[24] in realistic toroidal geometry. However, simple physical intuition and direct gyrokinetic particle simulations indicate that addition of external shear flows *reduces* turbulence spreading.[18] We argue that the reason the zonal flow might appear to play an important role in spreading is that fluctuation-fluctuation coupling is neglected in most of the models of turbulence-mean flow interactions. As a result, the zonal flow is, by construction, the *only path* through which nonlinear energy transfer may occur. This in turn creates the illusion that turbulence spreading is due to wave–zonal-flow interactions. We argue that for developed wave turbulence, the main cause of nonlinear spreading is the total contribution from direct interactions between fluctuations.

Note that, while zonal flows are probably not important (except to stop spreading), mean flows as represented in our theory by the use of mean PV [i.e., $\bar{q}(x)$], as well as the mean density fields [i.e., $\bar{n}(x)$] and local instabilities subsequently driven by the gradients of those, are important players for the phenomenon of penetrative convection.

In conclusion, we have constructed force-flux relations for the transport of turbulence intensity and calculated the transport matrix for fluctuation energy. The theory is cast in terms of wave interaction processes. We show that the evolution of the PV profile conveniently takes into account both the mean density and the mean flow evolution and yields a simple self-consistent model of turbulence transport involving a "pinch" of turbulence intensity.

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[1]H. D. Chen, S. A. Orszag, I. Staroselsky, and S. Succi, J. Fluid Mech. **519**, 301 (2004).
[2]X. Garbet, L. Laurent, A. Samain, and J. Chinardet, Nucl. Fusion **34**, 963 (1994).
[3]P. H. Diamond and T. S. Hahm, Phys. Plasmas **2**, 3640 (1995).
[4]P. H. Diamond and M. Malkov, Phys. Scr., T **T98**, 63 (2002).
[5]E.-J. Kim, P. H. Diamond, M. Malkov *et al.*, Nucl. Fusion **43**, 961 (2003).
[6]R. D. Sydora, V. K. Decyk, and J. M. Dawson, Plasma Phys. Controlled Fusion **38**, A281 (1996).
[7]S. E. Parker, H. E. Mynick, M. Artun *et al.*, Phys. Plasmas **3**, 1959 (1996).
[8]R. Nazikian, K. Shinohara, G. J. Kramer *et al.*, Phys. Rev. Lett. **94**, 135002 (2005).
[9]T. S. Hahm, P. H. Diamond, Z. Lin, K. Itoh, and S.-I. Itoh, Plasma Phys. Controlled Fusion **46**, A323 (2004).
[10]Ö. D. Gürcan, P. H. Diamond, T. S. Hahm, and Z. Lin, Phys. Plasmas **12**, 032303 (2005).
[11]T. S. Hahm, P. H. Diamond, Z. Lin, G. Rewoldt, O. Gurcan, and S. Either, Phys. Plasmas **12**, 090903 (2005).
[12]S. B. Pope, *Turbulent Flows* (Cambridge University Press, Cambridge, 2000), pp. 373–383.
[13]R. A. Fisher, Ann. Eugen. **7**, 353 (1937).
[14]J. D. Murray, *Mathematical Biology*, 2nd ed. (Springer-Verlag, Berlin, 1993).
[15]Ö. D. Gürcan, P. H. Diamond, and T. S. Hahm, Phys. Plasmas **13**, 052306 (2006).
[16]V. Naulin, A. H. Nielsen, and J. J. Rasmussen, Phys. Plasmas **12**, 122306 (2005).
[17]Ö. D. Gürcan, P. H. Diamond, and T. S. Hahm, Phys. Rev. Lett. **97**, 024502 (2006).
[18]W. X. Wang, Z. Lin, W. M. Tang, W. W. Lee, S. Ethier, J. L. V. Lewandowski, G. Rewoldt, T. S. Hahm, and J. Manickam, Phys. Plasmas **13**, 092505 (2006).
[19]Z. Lin, S. Ethier, T. S. Hahm, and W. M. Tang, Phys. Rev. Lett. **88**, 195004 (2002).
[20]Z. Lin and T. S. Hahm, Phys. Plasmas **11**, 1099 (2004).
[21]R. E. Waltz and J. Candy, Phys. Plasmas **12**, 072303 (2005).
[22]S. B. Korsholm, P. K. Michelsen, and V. Naulin, Phys. Plasmas **6**, 2401 (1999).
[23]A. Yoshizawa, Phys. Fluids **27**, 1377 (1984).
[24]L. Chen, R. B. White, and F. Zonca, Phys. Rev. Lett. **92**, 075004 (2004).